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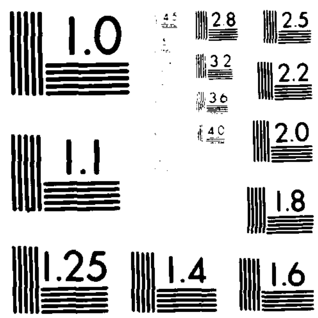

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A MONTE CARLO STUDY OF AR (1) ESTIMATORS UNDER SEVERAL PERFORMANCE CRITERIA.

by

(10) H. S. HILL

R. F. LING

Department of Mathematical Sciences

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ABSTRACT

The small sample performance of several AR(1) estimators is investigated through the use of Monte Carlo comparison studies. The performance of these estimators is compared with respect to the criteria of bias, mean squared error, mean absolute error, and mean squared prediction error. Statistical performance groupings at various fixed parameter values from (0,1) are determined based on pairwise multiple comparisons of estimator performance results.

Two types of two-step adaptive estimators are developed. One type relies on the use of only standard estimators, while the other type includes ad hoc modifications to standard estimators. The efficacy of performance of these estimators is validated through the use of additional Monte Carlo runs based on three different conditions of parameter selection for data generation. The sensitivity of these estimators to their use with larger sample sizes is also investigated.

Based on the various simulation results, recommendations regarding estimator selection for use in applied estimation are given. The applicability of the adaptive estimators is discussed and an example illustrating their application in forecasting an economic series is given.

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CHAPTER I

INTRODUCTION

A considerable amount of the econometric literature deals with studies involving economic time series. Since the publication of Box and Jenkins' book [3] which discusses forecasting with Autoregressive Integrated Moving Average (ARIMA) processes, an increasing number of studies have dealt with some aspect of ARIMA modeling and application.

The form of an ARIMA process is

$$x_t - \beta_1 x_{t-1} - \cdots - \beta_p x_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q} \quad (1.1)$$

where x_t is a continuous random variable with

$$\{x_t: t = 0, \pm 1, \pm 2, \ldots\}$$

being a discrete parameter time series, and

$$\{\varepsilon_t: t = 0, \pm 1, \pm 2, \ldots\}$$

is a sequence of independent and identically distributed random variables (shocks) with mean zero and variance σ^2. The β_i's are referred to as autoregressive parameters and the θ_i's as moving average parameters. In regards to ARIMA modeling, Box and Jenkins have said:

> The relating of a model of this kind to data is usually best achieved by a three stage iterative procedure based on identification, estimation, and diagnostic checking.
>
> By <u>identification</u> we mean the use of the data, and of any information on how the series was generated, to suggest a subclass of parsimonious models worthy to be entertained.

> By <u>estimation</u> we mean efficient use of the data to make inferences about parameters conditional on the adequacy of the entertained model. By <u>diagnostic checking</u> we mean checking the fitted model in its relation to the data with intent to reveal model inadequacies and so to achieve model improvement. [3], p. 171.

This dissertation only considers the estimation stage for one particular subclass of ARIMA models, the ARIMA process in which $p = 1$ and $q = 0$, which is referred to as an autoregressive process of order one, or simply AR(1). In this case (1.1) reduces to

$$x_t = \beta x_{t-1} + \varepsilon_t \ . \tag{1.2}$$

This dissertation deals primarily with problems surrounding parameter estimation and forecasting of AR(1) series. These problems are investigated through the use of Monte Carlo simulation studies. In these Monte Carlo simulations, series are generated by a process which is known to be stationary AR(1), i.e. with $|\beta|<1$. However, for the short series generated in this study, there is no guarantee that all of these series will appear stationary and exhibit clear AR(1) characteristics. Still no model identification or diagnostic checking are done on these simulated series, and adequacy of the AR(1) model for fitting the series is assumed. Possible influences of this assumption on the interpretation of the results will be discussed in Chapter VII.

Interest in the AR(1) model arises in the modeling of many economic series. The yearly and quarterly earnings of firms, real GNP, and consumer goods price index are among

the series which have been modeled as AR(1) series. The discussion of whether commodity futures prices and stock prices have any "structure" has centered around the question as to whether the series are random walks (AR(1) with $\beta = 1$). However, the researcher who identifies AR(1) models for short series is faced with problems in obtaining accurate parameter estimates. It is well-known that the ordinary least squares estimator is a biased estimator of β. For short series, the bias can be substantial. Thus, many alternative estimators have been proposed in an effort to overcome this bias problem. But it remains unclear as to which, if any, of these estimators provide adequate performance in the correction of bias, or whether such estimators are better in performance in terms of mean-squared error. In addition, if the objective is forecasting, other problems arise. Orcutt and Winokur [30] have shown that even if corrections are made for estimator bias, resultant predictive performance of the fitted models may be no better or even worse than the predictive performance of fitted models using the original biased estimator. This indicates that we are dealing with a situation where performance in terms of any individual criterion such as bias, mean-squared error (MSE), or mean-squared prediction error (MSPE) must be evaluated separately. Just as reducing the bias of an estimator does not guarantee a reduction in MSE, small bias and/or MSE does not of itself guarantee good predictive performance in terms of MSPE.

Previous investigation of these estimation problems seems to be rather incomplete. Consider, for example, the criterion of MSE. Several authors have done Monte Carlo studies comparing different sets of estimators. Conflicting results sometimes emerged because of the small number of replications used. For instance, Copas [9] found, using 100 replications of sample sizes of 10 and 20, that for $\beta < .6$, MSE of the maximum likelihood estimator was smaller than for ordinary least squares. But Thornber [36] found, for 100 replications of sample size 20, that MSE of ordinary least squares was noticeably smaller than for the maximum likelihood estimator for $\beta \geq .7$. Moreover, different parameters and sample sizes used in different studies make it difficult to make comparisons across studies. The same problem holds for the criteria of bias and MSPE.

Thus, we consider it appropriate to conduct a study encompassing a set of estimators that are either well-known or have been shown in the literature to be effective according to one of the three criteria (bias, MSE, and MSPE) and to conduct a sufficiently large scaled study to determine the relative effectiveness of those estimators according to each of the three criteria.

All of the previous Monte Carlo comparison studies have considered estimator performance at particular fixed values of β. Since no one estimator is best for all parameter values, this information is of little practical value in applied modeling where the value of β is certainly

unknown. Thus, there is a need to seek better estimators, estimators which exhibit better performance characteristics throughout a range of values of β, without having to assume any strong prior knowledge of the true value of β.

The resultant objectives of this research are the following:

(1) To investigate the small sample performance of several estimators through the use of a large-scaled Monte Carlo study;

(2) To make statistical comparisons of the results of these sampling studies;

(3) To use any information gained in the estimator comparison studies to aid in the development of estimation strategies for use in application as functions of the various estimation criteria.

In Chapter II, the estimators which are chosen for evaluation in this study will be stated with a summary of previous findings about some of them in the literature. The criteria of bias, mean absolute error, MSE, and MSPE will be defined. The empirical definitions of these quantities as used in the Monte Carlo studies will also be given. In Chapter III the design of the Monte Carlo simulations are discussed, including a brief discussion of the choice of sample size and selection of the number of replications. The results of estimator performance comparisons at fixed parameter values are given in Chapter IV. Estimator performance with regard to each of the four criteria of interest is discussed. Comparisons with the results of previous studies are made where possible. In Chapter V the proposed strategy for applied estimation is given. The

development of two types of adaptive estimators are outlined. Both types of adaptive estimators are based on a two-step procedure; i.e., obtaining a preliminary estimate at the first step, the value of which determines the choice of a particular estimator for use at the second step in obtaining a final estimate. The first type of adaptive estimators is based on the use of only standard estimators in the second step while the second type of adaptive estimators includes the use of some "ad hoc" modifications to standard estimators for use in the second step. Adaptive estimators of each type are constructed for the criteria of mean absolute error, MSE, and MSPE. The empirical determination of the adaptive estimators will be discussed in this chapter. In Chapter VI, the results of additional Monte Carlo runs which validate the effectiveness of the adaptive estimators are reported. Comparisons between the adaptive estimators and standard estimators are made under three different conditions of data generation. The first comparison is made for data sets generated with β drawn randomly from the interval (0,1). This condition of generation most closely simulates conditions of practical application where the true parameter is unknown, and one has only a vague idea about its true location. Additional estimator performance comparisons are made for sets of data generated with β drawn randomly from each one-tenth unit subinterval of the interval (0,1). Finally, comparisons are made for sets of data generated with the fixed β values considered in the comparisons discussed in

Chapter IV. In Chapter VII some of the limitations of the findings of this study are discussed. Included is a discussion of the sensitivity of the adaptive estimators to changes in sample size and in particular their applicability in estimating longer series. Chapter VIII concludes the dissertation with a discussion of the application of the results of this study. Based on the results of the estimator performance comparisons, recommendations for applied estimation are given. Some examples illustrating the potential areas of application of this research are discussed, and an example demonstrating the application of the adaptive estimators in forecasting an economic series is given.

CHAPTER II

BACKGROUND LITERATURE

Estimators Considered

Most of the eleven estimators considered in the Monte Carlo performance evaluations either appear in the time series literature or are mentioned in econometric texts. These include the least squares estimator and several of its modifications. With the exception of the maximum likelihood estimate, all of the estimates are based on combinations of sums of squares and sums of cross products and are therefore easy to obtain.

The first estimator considered is the ordinary least squares (OLS) or Gauss-Markov estimator. This estimator is referred to by Box-Jenkins as the "conditional" least squares estimator. It conditions on an initial value of the series which occurred prior to the observation period. This estimator is derived by the least squares principle of minimizing the sum of squared differences between observed and fitted values. That is, the estimator minimizes

$$SS(\beta) = \sum_{t=1}^{T} (x_t - \hat{x}_t(\beta))^2 , \qquad (2.1)$$

where $\hat{x}_t(\beta)$ is the fitted value. If we let

$$\hat{x}_t(\beta) = E[x_t | x_1, x_2, \ldots, x_{t-1}] = \beta x_{t-1} ,$$

then (2.1) becomes

$$SS(\beta) = (x_1-\beta x_0)^2 + \sum_{t=2}^{T} (x_t-\beta x_{t-1})^2 \quad . \tag{2.2}$$

If the unobservable initial value x_0 is set equal to its unconditional mean which is zero, minimization of the resulting sum of squares gives

$$b_1 = \frac{\sum_{t=2}^{T} x_t x_{t-1}}{\sum_{t=2}^{T} x_{t-1}^2} \quad . \tag{2.3}$$

The sampling distribution of b_1 is unknown. Hurwicz [19] showed that b_1 is a biased estimator of β and succeeded in evaluating the bias exactly for samples of size three. Asymptotic expansions for the mean and variance of b_1 were obtained by Marriott and Pope [24] and Kendall [21] and extended by White [38] and Shenton and Johnson [33]. The estimator b_1 is one of the estimators considered in the studies of Gonedes and Roberts [16], Dent and Min [10], Copas [9], Orcutt and Winokur [30], and Thornber [36].

The form of the OLS bias was approximated (to order 1/T) by some of the above authors to be $-2\beta/T$. Using this approximation as a correction term for b_1 results in the second estimator considered:

$$b_2 = [T/(T-2)] \cdot b_1 \quad . \tag{2.4}$$

This estimator was considered in Copas' [9] comparison studies, and a similar correction for OLS slope bias (where an intercept term was estimated from the data) was considered by Orcutt and Winokur [30].

The third estimator considered is sometimes referred to as the unconditional least squares estimator. If there is sufficient a priori reason to impose the constraint that the system be stationary, $|\beta| < 1$, then the marginal distribution of x_1 is given by:

$$x_1 \sim N(0, \sigma^2/(1-\beta^2)) .$$

Then the residual sum of squares would be written

$$SS(\beta) = (1-\beta^2)x_1{}^2 + \sum_{t=2}^{T}(x_t-\beta x_{t-1})^2 . \tag{2.5}$$

Minimizing (2.5) with respect to β gives

$$b_3 = \frac{\sum_{t=2}^{T} x_t x_{t-1}}{\sum_{t=3}^{T} x^2{}_{t-1}} . \tag{2.6}$$

Since (2.5) dominates the likelihood function of an AR(1), b_3 is also an approximation for the maximum likelihood estimator. This estimator is considered in the work of Thornber [36] and the comparisons of Dent and Min [10].

An idea given by Quenouille [31] for removing the bias of b_1 provides the basis for the fourth estimator considered. This method consists of calculating

$$b_4 = 2b_1 - ([b_1]_1 + [b_1]_2)/2 , \tag{2.7}$$

where $[b_1]_1$ is the OLS estimate for the first half of the sample data and $[b_1]_2$ the OLS estimate for the second half. Orcutt and Winokur [30] consider this estimator in their study.

The Yule-Walker estimator, given for example in Box and Jenkins [3], is

$$b_5 = \frac{\sum_{t=2}^{T} x_t x_{t-1}}{\sum_{t=2}^{T+1} x_{t-1}^2} . \qquad (2.8)$$

It is very similar in form to the least squares estimators b_1 and b_3. Since these three have the same numerator, it is easily seen by comparing the denominators that $|b_5| < |b_1| < |b_3|$. The Yule-Walker estimator was included in the comparison studies of Dent and Min [10].

The sixth estimator was suggested by K. Alam of Clemson University as a bias-correction modification to the OLS estimator (2.3). It takes on the form

$$b_6 = \frac{x_1 x_2 + \sum_{t=2}^{T} x_t x_{t-1} + x_{T-1} x_T}{\sum_{t=2}^{T} x_{t-1}^2} , \qquad (2.9)$$

which is

$$b_1 + \left[\frac{x_1 x_2 + x_{T-1} x_T}{\sum_{t=2}^{T} x_{t-1}^2}\right] .$$

Since

$$E\left[\frac{x_1 x_2 + x_{T-1} x_T}{\sum_{t=2}^{T} x_{t-1}^2}\right] = \frac{2\beta}{T-2} ,$$

this term affords a bias correction of very nearly the same magnitude as the $2\beta/T$ correction used in constructing estimator b_2.

For each of the first six estimators considered, the imposition of the stationarity condition, $|\beta| < 1$, on the generating process does not guarantee that these estimators will result in stationary estimates for any given short series. In this study, since all series are generated with $|\beta| < 1$, the resulting estimates were constrained to be less than or equal to one in absolute value by truncation, as is often done in practice and also was done in other Monte Carlo studies. This was accomplished by using the following simple truncation rule:

$$b_i = \left\{ \begin{array}{lll} b_i & \text{if} & |b_i| < 1 \\ +1 & \text{if} & b_i \geq 1 \end{array} \right\} \text{for } i = 1, 2, \ldots, 6.$$

Throughout the Monte Carlo estimator comparisons in this dissertation, these constrained versions of estimators one through six were used.

The next estimator considered is attributable to J. Burg and was reported by Foster [15]. The AR(1) case of Burg's estimator can be derived as a sum of squares minimization. Due to the symmetry of the joint distribution of $x_1, x_2, \ldots, x_T$, we see that the covariance structure of $\{x_t^*: t = 0, \pm 1, \pm 2, \ldots\}$, where $x_t^* = x_{T-t-1}$, is the same as the covariance structure of $\{x_t: t = 0, \pm 1, \pm 2, \ldots\}$. Hence, we can think of $x_1^*, x_2^*, \ldots, x_t^*$ as another (reversed) realization of the same series having sum of squared residuals of the same form as (2.2):

$$SS^*(\beta) = (x_1^* - \beta x_0^*)^2 + \sum_{t=2}^{T} (x_t^* - \beta x_{t-1}^*)^2 .$$

Substituting for x_i^* values, we get

$$SS^*(\beta) = (x_T - \beta x_{T+1})^2 + \sum_{t=2}^{T} (x_{t-1} - \beta x_t)^2 \ . \qquad (2.10)$$

If the unobservable x_{T+1} is set equal to its unconditional mean of zero, just as was x_0 in (2.2) when deriving the conditional least squares estimator in (2.3), then minimizing the average of $SS(\beta)$ in (2.2) and $SS^*(\beta)$ in (2.10) results in

$$b_7 = \frac{2 \sum_{t=2}^{T} x_t x_{t-1}}{x_1^2 + x_T^2 + 2 \sum_{t=3}^{T} x_{t-1}^2} \ . \qquad (2.11)$$

Review of the econometric literature revealed no use of Burg's estimator.

The eighth estimator considered is similar in form to the sample correlation coefficient. This estimator, given in Murphy [27] takes the form

$$b_8 = \frac{\sum_{t=2}^{T} x_t x_{t-1}}{\sqrt{\sum_{t=2}^{T} x_{t-1}^2 \cdot \sum_{t=2}^{T+1} x_{t-1}^2}} \ . \qquad (2.12)$$

The Durbin-Watson statistic (see [12]) forms the basis of the ninth estimator. Since this statistic takes on a value between zero and four, depending on the strength of the autocorrelation in the series, the form

$$b_9 = 1 - (D/2) \ , \qquad (2.13)$$

where

$$D = \frac{\sum_{t=2}^{T} (x_t - x_{t-1})^2}{\sum_{t=2}^{T+1} x_{t-1}^2} \tag{2.13}$$

is the Durbin-Watson statistic, takes on a value between negative one and one. Thus b_9 estimates the value of the autoregressive parameter. The estimator is given in Murphy [27] and in Johnston [20].

The tenth estimator considered is given by Maulinvaud [26], and takes the form

$$b_{10} = \frac{\sum_{t=2}^{T} x_t x_{t-1} - [x_1 x_T/(T-1)]}{\sum_{t=2}^{T+1} x_{t-1}^2} . \tag{2.14}$$

The estimators b_7, b_8, b_9, and b_{10} can all be shown to be less than one in absolute value. This guarantees that they give only stationary estimates for the autoregressive parameter.

The final estimator considered in the performance evaluation is the maximum likelihood estimator (MLE). The exact likelihood function for an AR(1) is given by

$$\begin{aligned} L(\beta,\sigma^2|\underline{x}) = {} & (2\pi\sigma^2)^{-T/2}[1/(1-\beta^2)]^{-1/2} \\ & \exp\{[-1/(2\sigma^2)] \cdot SS(\beta)\} \end{aligned} \tag{2.15}$$

where $SS(\beta)$ is the sum of squares given in (2.5). Substituting the maximizing value of σ^2 for a given value of β into (2.15) will produce a likelihood function that is independent

of σ^2 (see [4]), namely

$$L(\beta|\underline{x}) = (2\pi e/T)^{-T/2}[1/(1-\beta^2)]^{-1/2}[SS(\beta)]^{-T/2} . \tag{2.16}$$

Finding b_{11}, the value of β which maximizes (2.16), is equivalent to finding the value of β which minimizes

$$SS(\beta) \cdot [1/(1-\beta^2)]^{1/T} . \tag{2.17}$$

The function in (2.17) must be minimized over the interval (-1, 1) through the use of some numerical technique. For our purposes a simple though rather crude search routine was used. This was the iterative three point evaluation, interval bisection method used by Clawson [7].

Comparison Criteria

In order to adequately evaluate the overall performance of the eleven estimators discussed above, four criteria for comparison were chosen. These were bias, mean absolute error, mean squared error (MSE) and mean squared prediction error (MSPE). Several comparison studies have considered bias. Therefore, for comparison with these studies this criterion was included. For any given estimator $\hat{\beta}$, the bias in estimating a parameter β is defined to be

$$\text{Bias}(\hat{\beta}) = E[\hat{\beta} - \beta] .$$

The empirical bias of an estimator $\hat{\beta}$, as computed for parameter value β in each Monte Carlo run, is defined to be

$$\sum_{i=1}^{k} (\hat{\beta}_i - \beta)/k$$

where k is the number of replications performed in the run. Since positive and negative sample errors tend to cancel

each other out, mean absolute error was felt to be a more revealing criterion of location accuracy than bias. The mean absolute error of an estimator $\hat{\beta}$ is defined to be

$$\text{Mean Absolute Error}(\hat{\beta}) = E[|\hat{\beta} - \beta|] .$$

The mean absolute error of an estimator for a Monte Carlo run is computed as

$$\sum_{i=1}^{k} |\hat{\beta}_i - \beta|/k$$

for k replications. The third estimation criterion compared was that of mean squared error. The mean squared error of estimator $\hat{\beta}$ is defined to be

$$\text{MSE}(\hat{\beta}) = E[(\hat{\beta} - \beta)^2] .$$

The empirical MSE is computed as

$$\sum_{i=1}^{k} (\hat{\beta} - \beta)^2/k$$

for k replications. In order to compare forecasting accuracy using the various estimates, mean squared prediction error was compared. The one-step-ahead mean squared prediction error associated with estimator $\hat{\beta}$ is given by

$$\text{MSPE}(\hat{\beta}) = E[(\hat{x}_t - x_t)^2]$$

where $\hat{x}_t = \hat{\beta}x_{t-1}$. The empirical one-step MSPE in the forecasting of one series is given by

$$\sum_{t=1}^{m} (\hat{x}_t - x_t)^2/m ,$$

where m is the number of one-step-ahead post-sample predictions made. For each series, MSPE of a given estimator was

computed as the average squared error in making twenty one-step-ahead predictions for that particular series where the parameter estimate used in the prediction was computed using the given estimator. The MSPE reported in this study is the empirical MSPE averaged over all one-step predictions and all replications, namely

$$\sum_{i=1}^{k} [\sum_{t=1}^{m} (\hat{x}_t - x_t)^2/m]/k \ .$$

Previous Simulation Studies

Previous estimator comparison studies which considered AR(1) parameter estimation have compared only a few of the eleven estimators considered in this study. They generally reported comparison results on the criteria of bias and/or MSE at fixed β values. They all differ considerably as to which β values were chosen, what sample sizes were considered, and how many replications were performed. In Table I, some references to the estimators considered in this study are listed. References to previous simulation studies which included some of these estimators are given by estimator and criterion of consideration. The sparseness of entries in Table I illustrates the need for a comprehensive comparison of these estimators.

For AR(1), Copas [9] compared the performance of mean likelihood, OLS, MLE, and sample autocorrelation estimators. He considered the one-parameter model (1.2) with sample sizes n = 10 and 20, for 100 replications. His

Table I. Summary of Estimator References for This Study and References to Previous Simulation Studies Which Have Included These Estimators.

Estimator	General	Bias	MSE	MSPE
1	[3,20,26,27]	[9,10,30]	[9,10,16,30,36]	[16,30]
2	[20,21,24]	[21,24,30]	[30]	[30]
3	[3,36]	[10]	[10,36]	
4	[30,31]	[30,31]	[30]	[30]
5	[3]	[10]	[10]	
6				
7	[15]			
8	[27]			
9	[12,20,27]			
10	[26]			
11	[3,4,9,10,36]	[9,10]	[9,10,16,36]	[16]

simulations considered two approaches for starting series generation:

(1) $x_1 = 1$

(2) $x_1 \sim N(0,1/(1 - \beta^2))$.

For $x_1 = 1$ studies he used $\beta = -.9(.1).9$, $n = 10$, and for model (2) used $\beta = -.9(.1).9$ for $n = 20$ and $\beta = -.8(.2).8$, $n = 20$. He compared on the criteria of bias and MSE. His conclusions were that mean likelihood gave the smallest MSE in the range (0,.6), OLS being better for $\beta > .6$ for initial value (1). For initial value (2), MLE was slightly better than OLS for $\beta > .6$. (This disagrees with results of Thornber and of Dent and Min who found MLE performance under condition (2) to be not so good in (.5,1)).

Thornber [36] considered the AR(1) model with one-parameter. He investigated the performance of

(1) OLS ("conditional" least squares),

(2) unconditional least squares,

(3) MLE, and

(4) Bayesian minimum expected loss.

He considered ten values of β on the interval (0,1), using 100 replications of sample size 20. His results showed that MSE was smaller for MLE over (0, .5), but Bayesian and conditional least squares (truncated to one if greater than one) had the smallest MSE over (.5,1).

Orcutt and Winokur [30] studied two-parameter AR(1), i.e.,

$$x_t = \beta_0 + \beta_1 x_{t-1} + \varepsilon_t \quad . \tag{2.18}$$

They showed the extent of bias in OLS estimation of the slope parameter for 1000 replications on samples of size 10, 20, and 40 for β values of -1(.25)0, and 0(.1)1.1. They found OLS estimates of β negatively biased for β > -0.5 with increasing bias as sample size was decreased and β increased from -0.5 to 1.0. They considered two bias-corrected estimators based on corrections given by Marriot and Pope [24] and Quenouille [31] (similar to our estimators b_2 and b_4 but for the two-parameter case), which they compared with OLS and MSE. They found both modifications were essentially unbiased for β values of 0,.3,.6,.9,1.0 and sample sizes 10, 20, and 40. For smaller β values, the OLS estimator, though biased, still had the lowest MSE. For large β, the Marriott and Pope correction had smaller MSE. In considering predictive performance, they reported on 2, 3, and 4 period predictions. They found the OLS fitted model predicted better (smaller prediction error variance) than either of the bias-corrected estimation models.

Gonedes and Roberts [16] studied the two-parameter AR(1) with emphasis on estimator performance for nearly non-stationary series. They concluded that if sample size is small and if good one-step prediction is the goal, then one should difference the data (as if it were non-stationary) and treat the differences as a stationary AR(1). They did simulations by generating data from stationary AR(1) (with $\mu = 0$) and then studying estimation and prediction by

(1) Comparing OLS, mode of joint posterior density of μ, β, and σ, and random walk without drift on the original series;

(2) Comparing OLS, modal, and random walk on series of first differences.

They considered $\beta = 0$ .2,.5,.7,.9,.99 for sample sizes $n = 20$, 30, and 60, based on 50 replications. For $\beta \geq .7$, MSE was always less for the modal estimator than for OLS. For $\beta < .7$, MSE was nearly the same for the two while OLS had a slight edge. For 20 one-step predictions, the modal estimator showed a slight edge over OLS in MSPE for larger β and smaller n, but the margin narrowed rapidly as β was reduced or n was increased. However, for $\beta \geq .9$, for all n, MSPE for random walk was substantially lower than for OLS or modal. This was also the case for OLS and modal on first differences. However, differenced models were much better than undifferenced and were only slightly outperformed by random walk.

Dent and Min [10] considered six ARMA models with respect to properties of a variety of proposed estimators. These mainly involved MLE and least squares estimators. For the AR models they compared:

(1) Unconditional least squares

(2) Conditional least squares

(3) Yule-Walker estimator

(4) Approximate maximum likelihood

(5) Exact maximum likelihood

(6) Kendall's estimator (based on higher-order sample autocorrelations)

(7) Quenouille's estimator (also based on higher-order sample autocorrelations)

They used sample sizes of n = 100 throughout, with 100 replications. For AR(1) they considered twelve β values ($\pm.9$, $\pm.7$, $\pm.6$, $\pm.5$, $\pm.3$, $\pm.1$) for generating data and compared estimator performance on the criteria of bias and MSE. Their general conclusions were the following: for AR(1) little differentiation could be made between estimators considered; bias tended to be negative for all estimators with Quenouille having minimum absolute bias at positive β, while MSE was minimum for exact MLE (0, .5) or unconditional least squares (.5, 1).

CHAPTER III

SIMULATION DESIGN

In making estimator comparison studies and developing adaptive estimators, many Monte Carlo simulation runs were made. A run of the simulation consisted of the generation of a large number of replications and the computation of summary statistics. These summary run statistics consisted of averages and variances by estimator and parameter value over all replications in a run. Each replication included the generation and parameter estimation of one series for each parameter value considered.

For these Monte Carlo simulations, a sample size (series length) of 20 was chosen, since we are interested in estimator performance when considering samples as small as would likely be encountered in the study of economic series. Also, in order to compare the results with the results of previous studies, 20 seemed to be most appropriate since it was the most commonly used sample size.

For each replication of a simulation run, 40 standard normal shocks were generated for use in the construction of one series for each parameter value. These standard normal variates were generated using a method of Marsaglia and Bray [25] using the uniform (0,1) variates generated by the pseudo-random number generator given by Lewis, Goodman, and Miller [23]. The particular generator described by them is

$x_{i+1} = Ax_i$ (mod p) where $p = 2^{31}-1$ and $A = 16807$. The authors cite numerous test results substantiating the effectiveness of the generator.

Since all series were generated under the assumption of stationarity, $|\beta| < 1$, which implies that each

$$x_i \sim N(0, \sigma^2/(1 - \beta^2)) ,$$

an initial observation can be considered to be

$$x_1 \sim N(0, 1/(1 - \beta^2)) .$$

For a given parameter value, an initial value was generated, and, using model (1.2), $x_t = \beta x_{t-1} + \varepsilon_t$, an AR(1) series of length 40 was generated. Using the first 20 terms of this series as the sample data, sums of squares and cross products were computed, each estimator evaluated, and the values accumulated. Any estimates greater than one were truncated to one. Then, for the next parameter value, an initial value was generated, a series generated, and estimators evaluated. Once this was done for each parameter value considered, the entire procedure was repeated for each additional replication. Here, parameter values of .1(.1).9, .95, .99, and .999 were selected. These parameter values were chosen to provide representation throughout the positive range of stationarity with some emphasis on performance near the boundary of stationarity.

The measure of MSPE of an estimator for one series consisted of the average squared misses of twenty one-step-ahead predictions when using that estimator. This was the

reason for initially generating series of length 40. As discussed above, the first 20 observations of the series were used for obtaining parameter estimates. Then using the fitted model, and the last observation of the 20, the 21st term was forecast. The square of the amount by which this predicted value of the 21st term missed the actual 21st term of the series was the first component of the sum of the squared prediction error. The second one-step prediction used the actual 21st term of the generated series along with the same fitted model for forecasting the 22nd term, etc. This procedure was repeated 20 times for each estimator on each series.

At the end of each run of the simulation, several quantities were computed and a table output for each estimator at each parameter value considered. These quantities included the empirical form of bias, mean absolute error, MSE, and MSPE as defined in Chapter II. One such table is shown as Table II, which is the output for estimator 9 from one simulation run of 10000 replications. In Table II, the first column shows the 12 parameter values used. The next two columns show the average and variance of the 10000 parameter estimates for each parameter value. Columns 4 and 5 show the average and variance of $(\hat{\beta} - \beta)$ over 10000 replications. For example, for $\beta = .1$,

$$0.03863 = \sum_{i=1}^{10000} (\hat{\beta}_i - 0.1)/10000 ,$$

Table II. Sample of Output From Monte Carlo Runs Showing Empirical Means and Variances for Estimator 9.

BETA	E(B-HAT)	VAR(B-HAT)	E(BIAS)	VAR(BIAS)	E(|ERR|)	VAR(|ERR|)	E(BIAS**2)	VAR(BIAS**2)	E(MSPE)	VAR(MSPE)	TRUNC
.100	0.13863	0.04468	0.03863	0.04468	0.17400	0.01589	0.04616	0.00364	1.04569	0.11840	0
.200	0.22375	0.04278	0.02375	0.04278	0.16847	0.01496	0.04334	0.00332	1.04387	0.11764	0
.300	0.31055	0.04070	0.01055	0.04070	0.16271	0.01434	0.04081	0.00313	1.04294	0.11842	0
.400	0.39720	0.03819	-0.00280	0.03819	0.15625	0.01378	0.03819	0.00310	1.04368	0.11935	0
.500	0.48290	0.03528	-0.01710	0.03528	0.14832	0.01357	0.03557	0.00316	1.04462	0.12232	0
.600	0.56932	0.03181	-0.03068	0.03181	0.13965	0.01324	0.03274	0.00324	1.04806	0.12703	0
.700	0.65836	0.02703	-0.04164	0.02703	0.12713	0.01260	0.02877	0.00320	1.05171	0.13578	0
.800	0.74650	0.02229	-0.05350	0.02229	0.11318	0.01234	0.02515	0.00317	1.06304	0.16072	0
.900	0.84065	0.01522	-0.05935	0.01522	0.09036	0.01058	0.01874	0.00231	1.07883	0.21405	0
.950	0.89180	0.01111	-0.05820	0.01111	0.07365	0.00907	0.01450	0.00174	1.09603	0.31019	0
.990	0.95072	0.00593	-0.03928	0.00593	0.04256	0.00566	0.00747	0.00086	1.08184	0.32667	0
.999	0.98427	0.00198	-0.01473	0.00198	0.01508	0.00197	0.00220	0.00024	1.02738	0.17482	0

and

$$0.04468 = \sum_{i=1}^{10000} [(\hat{\beta}_i - 0.1) - 0.03863]^2/10000 .$$

Similarly, columns 6 and 7 show the mean and variance of $|\hat{\beta} - \beta|$, and columns 8 and 9 the mean and variance of $(\hat{\beta} - \beta)^2$ over 10000 replications. Results for MSPE averaged over all one-step predictions and all replications are given in columns 10 and 11. The final column shows the number of estimates truncated to one, which for estimator 9 was always zero.

For estimator performance comparisons, various averages such as those illustrated in Table II from 10000 replications were used. This large number of replications was necessary to ensure sufficiently small standard errors of point estimates so that the effectiveness of various estimators could be statistically discriminated. Some preliminary runs showing averages using sets of 1000 and sets of 5000 replications indicated the need for a larger number of replications.

For example, when two different sets (different generator seeds) of 1000 replicates each were run, the typical difference in MSE between two estimators was in the 0.001 to 0.003 range, which is roughly of the same order of magnitude as the differences in MSE of the same estimator between sets of 1000 replicates. For example, the MSE for b_1 with $\beta = .1$ was found to be 0.04662 on one run and 0.04973 on the second - a difference of 0.00311. Similarly, the MSE for b_{11} at

β = .1 was 0.04665 and 0.05032 respectively, so that the difference between estimators b_1 and b_{11} was smaller in each case than the 0.00311 difference between runs for b_1. Also, for only 1000 replications there were numerous instances where the rank order of estimator performance was not preserved between sets of replicates, especially for β in the range of 0.7 to 0.9. Of course, the same lack of clear separation between estimators occured with MSPE. Differences between runs for the same estimator frequently were in the 0.01 to 0.025 range while several estimators within runs differed by less than 0.01.

CHAPTER IV

ESTIMATOR PERFORMANCE COMPARISONS

In this chapter, comparisons of performance of the eleven estimators are considered. The performance with regard to each of the four criteria of bias, mean absolute error, mean squared error (MSE), and mean squared prediction error (MSPE) is discussed for each of the parameter values considered. In printing our initial simulation results, one table was given for each estimator. One of the eleven such tables is shown as Table II. The contents of this table were discussed in Chapter III.

In order to compare these results by parameter value, numerous tables of statistical multiple comparisons were computed. One such table is shown as Table III. Each of these tables indicates estimator performance similarities and differences for one of the criteria at a given parameter value. The entries in Table III are t values computed for comparing each pair of means. Since the sample sizes are so large, the t's can be compared to a critical value of the standard normal z. Here Bonferroni's method was used to find this critical value for the 55 simultaneous tests of equality of means in each table. For a test at the $\alpha = .01$ significance level, the critical value is the $(1 - .01/110)$ or 0.999909 fractile of the standard normal distribution, which is approximately 3.75. Thus, we can see

Table III. Simultaneous Pairwise Comparisons of Empirical MSE of 11 Standard Estimators for $\beta = .9$.

ESTIMATOR	6	9	3	2	11	7	4	1	8	5
9	0.0[a]									
3	0.028	0.029								
2	0.557	0.576	0.531							
11	3.183	3.278	3.165	2.685						
7	5.198*	5.347*	5.186*	4.730*	1.995					
4	5.396*	5.541*	5.384*	4.942*	2.279	0.332				
1	7.376*	7.583*	7.369*	6.934*	4.123*	2.111	1.728			
8	10.560*	10.838*	10.559*	10.158*	7.286*	5.277*	4.826*	3.192		
5	16.399*	16.799*	16.410*	16.059*	13.048*	11.021*	10.456*	8.956*	5.732*	
10	16.411*	16.811*	16.422*	16.072*	13.065*	11.041*	10.476*	8.978*	5.758*	C.032

[a] values are t statistics computed from data such as that given in Table II.

*Significant at the $\alpha = 0.01$ level by Bonferroni's method of simultaneous comparison.

in Table III that each of the estimators (6, 9, 3, 2, 11) has significantly smaller MSE than that of each of the estimators (1, 8, 5, 10). In fact each of the estimators (6, 9, 3, 2) has significantly smaller MSE than each of the estimators (7, 4, 1, 8, 5, 10). These multiple comparison results are summarized in Table A-I - Table A-IV. These tables give performance ranking of each estimator, along with statistical class groupings of equivalent performance for each criterion at each parameter value. In these tables, any two estimator numbers not underscored by the same line have significantly different means for the given criterion at the given parameter value. Any two estimator numbers underscored by the same line are not significantly different. The estimator numbers in each case are arranged from left to right in ascending order of the criterion averages.

Based on the multiple comparison results, Tables IV and V were constructed by placing some estimators in a "best performance" group and some in a "worst performance" group for each criterion. For each set of parameter values, the estimators in the same group are usually not statistically different in their performance, while each estimator in the "best" group is significantly better than each estimator in the "worst" group. This categorization of estimator performance emphasizes several patterns and trends which are present.

Let us consider Table IV showing best and worst bias performance results. In general, there is a great deal of

Table IV. Best and Worst Estimator Performance Groups at Fixed β Values for Criterion of Bias.

	β Values					
	.1	.2	.3	.4	.5 & .6	>.6
Best Bias	4,3,2,6,11 1,7,8,10,5	6,2,4,3	2,6,4,9,3	9,2,6,4	6,2,4	6,2
Worst Bias	9	8,9,10,5	8,10,5	8,5,10	10,5	5,10

inconsistency in performance for ($.1 \leq \beta \leq .4$) with much more consistent results for $\beta \geq .5$. Estimators 2 and 6, which incorporate bias correction terms to OLS, seem to perform well throughout the parameter range. In fact, estimator 6 dominates all other estimators for $\beta \geq .5$.

In Table V, which shows best and worst estimators in terms of mean absolute error, MSE, and MSPE, there are several apparent patterns. Performance inconsistencies within a criterion occur mainly at $\beta = .7$, .8, and .9. Criteria are very similar in terms of best and worst performance groups at all parameter values except $\beta = .7$, .8, and .9. Estimator 9 appears to exhibit the best overall performance. It shows the smallest mean absolute error, MSE, and MSPE for ($.3 \leq \beta \leq .9$) and is significantly better than all other estimators for ($.5 \leq \beta \leq .8$). Also for $\beta = .95$ and $\beta = .999$ it ranks as the best estimator of the ones which do not generate any non-stationary values.

The fact that in our simulations any estimate greater than one is truncated, explains the prominence of estimators 2, 3, and 6 for $\beta > .9$. For the larger parameter values, the large number of these truncations has a dominant influence on the performance values. For instance, for estimator 2 at $\beta = .9$ there were 3758 truncations out of the 10000 estimates; at $\beta = .95$ there were 5706; at $\beta = .99$ there were 8174; and at $\beta = .999$ there were 9469. There were very nearly the same number in each case for estimator 6 and only slightly fewer for estimator 3. Obviously for these

Table V. Best and Worst Estimator Performance Groups at Fixed β Values for Criteria of Mean Absolute Error, MSE, and MSPE.

	β Values					
	<.4	.4,.5 & .6	.7	.8	.9	>.9
Best						
Mean Abs. Error	5,8,9,10	9	9	9	9,11	2,3,6
MSE	5,8,9,10	9	9	9	2,3,6,9,11	2,3,6
MSPE	5,9,10	9	9	9	2,3,6,9	2,3,6
Worst						
Mean Abs. Error	2,4,6	2,4,6	2,4,6	2,4,5,6,10	5,10	5,10
MSE	2,4,6	2,4,6	2,4,5,6,10	4,5,8,10	5,10	5,10
MSPE	4,6	2,4,6	2,4,6	4,5,6,10	5,10	5,10

estimators, as the true parameter value got closer to one, while at the same time a large percentage of the estimates became one, the performance appeared to be very good. This same truncation influence occurs with estimators 1 and 4, but they each have considerably fewer truncations than do estimators 2, 3, and 6.

We can notice that estimators 2, 4, and 6 are significantly worse than all others throughout ($.1 \leq \beta \leq .6$), and probably only appear much better for $\beta > .7$ due to the influence of the truncations discussed above. This is in contrast to the good performance in terms of bias observed for these three estimators in Table IV.

It is also interesting to note that estimators 5, 8, and 10 appear in the best performance class (along with estimator 9) or second only to estimator 9 for ($.1 \leq \beta \leq .6$), and then rapidly drop to becoming the worst in performance as β approaches one. In fact, for $\beta \geq .9$ estimators 5 and 10 constitute the worst class. This performance seems to relate to the poor performance of these estimators in terms of bias as observed in Table IV. Two of the more common traditional estimators, OLS (estimator 1) and MLE (estimator 11), exhibit only moderate performance throughout.

The only direct comparisons with previous studies which can be made are the studies of Copas [9], Thornber [36], and Dent and Min [10], all of whom investigated the one-parameter AR(1). All three of the above studies investigated OLS (constrained to be less than or equal to one) and MLE.

Copas found the differences in bias and MSE between these two estimators to be small and nearly constant over β with MLE having a uniformly lower MSE than OLS. Dent and Min found MSE performance of OLS to worsen relative to MLE as β increased. In contrast to this, Thornber found MLE to have smaller MSE over (0,.5) and larger MSE for $\beta > .6$. We find almost no difference in the bias of these two estimators at all parameter values considered. We also find almost no difference in the MSE performance of these two estimators for $(.1 \leq \beta \leq .5)$ (with neither being dominant), but find MLE to have a slight edge for all $\beta \geq .6$.

Thornber, and Dent and Min also investigated unconditional least squares. They both found that neither unconditional least squares nor MLE dominated the other with respect to MSE. They both found the sample loss functions of the two estimators to cross, with MLE having smaller MSE for small β, and unconditional least squares having smaller MSE for larger β. We also find the sample loss functions of these two estimators to cross in the same manner. The only differences seem to be in where they cross. Dent and Min found them to cross at $\beta = .5$; Thornber found them to cross between $\beta = .6$ and $\beta = .7$; and we find them to cross between $\beta = .7$ and $\beta = .8$.

Even though the investigations of Orcutt and Winokur [30] were with the two-parameter AR(1), we can compare with one of their findings. In comparing OLS, and two bias corrections to OLS, they found that the bias-corrected

estimators did not perform as well as OLS in terms of MSPE. Similarly, Copas found the MSE of a bias-corrected OLS estimator to be larger than the MSE of OLS. We see from our study that bias-corrected estimators 2, 4, and 6 all perform very poorly in terms of MSPE throughout most of the parameter range.

The overall results of these estimator comparisons indicate the need for additional investigation. The fact that the groups of best and worst estimators changed considerably over different values of the parameter range point out the need for some sort of selection strategy which, for any given problem, selects an appropriate estimator from among several estimator candidates, as opposed to always using some particular one estimator. In Chapter V, the attempts at development of such an estimation strategy will be discussed.

CHAPTER V

DEVELOPMENT OF ADAPTIVE ESTIMATORS

The Need for an Applied Estimation Strategy

The information about estimator performance at fixed parameter values gained from the Monte Carlo study is of little direct benefit in any applied estimation or forecasting problem. Unless the researcher has some strong a priori knowledge about the location of the true parameter value upon which to base the selection of an estimator which performs well for such a parameter value, he does not have a basis for choosing between available estimators. Of course, based on the results presented in Chapter IV, one would probably choose estimator 9 because of its good performance at a number of the parameter values studied. However, since estimator 9 was not best for each criterion at all parameter values, it was felt that an estimation strategy which incorporated the use of several estimators might perform better overall than any individual estimator such as estimator 9. In this chapter we discuss the development of such an estimation strategy.

Construction of Adaptive Estimators

The applied estimation strategy developed here uses a two-step estimation process. A preliminary estimate is obtained at the first step. For a given criterion, the value of this estimate determines the choice of a

particular estimator for use at the second step in obtaining a final estimate. Two types of such adaptive estimators were developed in this study. The first type of adaptive estimators is based on the use of only standard estimators on the second step, while the second type of adaptive estimators includes the use of some "ad hoc" modifications to the standard estimators for use in the second step. The maximum likelihood estimator was not included due to its computational difficulty relative to the other estimators considered. Adaptive estimators were developed for the criteria of mean absolute error, MSE, and MSPE.

The development of these adaptive estimators was dependent upon two requirements: the choice of an estimator for use in obtaining the preliminary estimate, and a set of rules for selecting the final estimator based on the value of the preliminary estimate and the estimation criterion of interest. In this study, estimator 7 was chosen as the preliminary estimator. This choice was based primarily on the fact that estimator 7 does not yield any non-stationary estimates, and it has consistently good MSE performance for all parameter values. Obviously, based on the results discussed in Chapter IV, estimator 9 would appear to be a logical choice. However, much preliminary work was done using estimator 7 before estimator 9 came to our attention. Since estimator 9 was later incorporated into the set of second-step estimators, it remains unclear as to whether the reconstruction of the adaptive estimators using estimator 9 as the preliminary estimator would offer any overall improvement.

An empirical determination of the rules for selection of second-step estimators was accomplished through the use of additional Monte Carlo runs. An initial run was used to determine series "similarities" based on the step-one estimate for a series. Different randomly generated series were grouped together if their estimation resulted in very similar values for estimator 7. This was accomplished in the following manner. First, a value of β was chosen randomly from the interval (0,1). Using this value of β, an AR(1) series of length 20 was generated in the same manner as described previously. The estimate b_7 was computed for this series. Depending on the value of b_7, this series was assigned to one of eleven "cells". These "cells" are subintervals of the parameter range, namely (-1,0), (0,.1), (.1,.2), (.2,.3), ..., (.8,.9), (.9,1). This process was repeated until 10000 series had been assigned to each cell.

A cell by cell estimator performance analysis was then made. For the first type of adaptive estimators, this meant the same type performance analysis runs as for the study discussed in Chapter IV. It consisted of determining which standard estimator had the best performance in each cell for each of the criteria of interest. The results of these runs are summarized in Table VI, which actually defines the first type of adaptive estimators. This table illustrates, for instance, that in the application of the first type of adaptive estimator, if the estimate b_7 is 0.38, and MSE is the criterion of interest, then estimator b_3 should be used for obtaining a final estimate.

Table VI. Definition of First Type of Adaptive Estimators (Al) Showing Second-Step Estimators for Use in Each Cell.

Cell	b_7-range	Mean Abs. Error	MSE	MSPE
1	(-1,0.)	b_9	b_9	b_9
2	(0.,.1)	b_9	b_9	b_9
3	(.1,.2)	b_9	b_9	b_9
4	(.2,.3)	b_9	b_9	b_9
5	(.3,.4)	b_3	b_3	b_9
6	(.4,.5)	b_7	b_7	b_3
7	(.5,.6)	b_7	b_7	b_3
8	(.6,.7)	b_7	b_7	b_3
9	(.7,.8)	b_7	b_7	b_9
10	(.8,.9)	b_7	b_7	b_9
11	(.9,1.)	b_7	b_7	b_7

In an effort to improve on the performance of the first type of adaptive estimators, considerable cell-by-cell investigation was done in search of "ad hoc" modifications to standard estimators which would improve their performance in terms of one or more criteria in some particular cell. In some cases ad hoc modifications which gave improved performance were functions of the estimator being modified. In other cases these modifications were just constants. These constants were usually related to the size of the bias observed for some estimator in a particular cell. In most instances these modifications were constructed in a manner so as to reduce bias in a particular cell without causing enough of an increase in variance to allow for a net reduction in mean absolute error or MSE.

Numerous modifications of various forms were considered. Modifications were made primarily to estimators 7 and 9 since they both had a small variance within each cell and also gave only stationary estimates. Initially, terms of the form $(1-b_i)^2/c$ were added to estimator b_i, where c is a positive integer. This form was chosen for several reasons. For one, the resulting estimator was still stationary. Since the term $(1-b_i)^2/c$ inflated the estimate b_i, yet still resulted in estimates less than one, the resulting variance of the modified estimator was smaller. Also, for cases where the bias of b_i was negative, the added term served to reduce this negative bias. After preliminary investigations showed that this term was too small as a

modification for large b_i values, the very similar form $(1-b_i^2)/c$ was adopted. Again, for $c \geq 2$, the addition of this term still resulted in stationary estimates with the desirable properties discussed above.

Investigating the performance of the estimator $b_7 + (1-b_7^2)/c$ for values of c between 2 and 10 indicated that for large values of b_7, c needed to be small; while for smaller values of b_7, c needed to be larger. Thus for some of the lower cells the estimator $b_7' = b_7 + (1-b_7^2)/10$ proved to be an ad hoc estimator which resulted in better performance for some criteria. In order to generalize the selection of c for use in different cells, the selection of c was tied to the size of the estimate b_7, by letting $c = c^*$ where $c^* = 10\{1 - ([10 \cdot b_7]/10)\} + 1$, which resulted in c taking on values between 2 and 11. The resulting estimator $b_7^* = b_7 + (1-b_7^2)/c^*$ was another ad hoc estimator which was used successfully in some cells with improved performance for some criteria. Since the bias of estimators 7 and 9 was positive in the upper cells, estimators b_7' and b_7^* and similarly modified b_9 did not always show improved performance for any criterion in these cells, as the addition of the modifying terms resulted in extremely large positive bias. In an effort to correct for this positive bias, even though variance was usually worsened, several estimators were investigated which were constructed by subtracting terms of the form described above. One such ad hoc estimator which performed well in the upper cells was the estimator $b_9^* = b_9 - (1-b_9^2)/10$.

Even though many ad hoc estimators of the form above were investigated, the only three of these estimators used as second-step estimators in the final ad hoc adaptives were b_7', b_7^*, and b_9^*. Other second-step estimators included these or standard estimators adjusted by the addition or subtraction of a constant. For some criteria in some cells, neither type modification resulted in better performance than the best standard estimator. In that case, the best standard estimator was used as the second-step estimator.

The resulting set of second-step estimators for each cell which make up the ad hoc type adaptive estimators are given in Table VII. In order to illustrate the choice of these ad hoc estimators for a representative cell, let us consider cell 4. The results of the cell 4 performance analysis for the ten standard estimators and for three ad hoc estimators, giving means and variances of parameter estimates, and empirical bias, mean absolute error, MSE, and MSPE are given in Table VIII. In Table VIII we see that the smallest mean absolute error, MSE, and MSPE of the ten standard estimators is exhibited by estimator 9. We can notice that b_7^* has in fact over-corrected for the negative bias of b_7, resulting in a substantial positive bias but with a slightly smaller variance. It is also noteworthy that b_7^* shows a smaller MSPE than all of the standard estimators, in spite of its positive bias. To eliminate this bias of +0.04495, this quantity was subtracted as a constant adjustment to b_7^* . This resulted in a mean

Table VII. Definition of Second Type of Adaptive Estimators (A2) Showing Second-Step Estimators for Use in Each Cell.

Cell	b_7-range	Mean Abs. Error	MSE	MSPE
1	(-1,0.)	$b_7^* + 0.2$	$b_7^* + 0.21679$	$b_7^* + 0.21679$
2	(0.,.1)	$b_7^* + 0.06853$	$b_7^* + 0.06853$	$b_7^* + 0.1$
3	(.1,.2)	b_7'	$b_7^* + 0.00502$	$b_7^* + 0.00502$
4	(.2,.3)	$b_7^* - 0.04495$	$b_7 - 0.04495$	$b_9 + 0.05$
5	(.3,.4)	b_3	b_3	b_7'
6	(.4,.5)	$b_9 - 0.02$	$b_9 - 0.02$	b_3
7	(.5,.6)	$b_9 - 0.03$	$b_9 - 0.03$	b_3
8	(.6,.7)	$b_9 - 0.03$	$b_9 - 0.03$	b_3
9	(.7,.8)	$b_9 - 0.03$	$b_9 - 0.03$	b_9
10	(.8,.9)	$b_9 - 0.03$	$b_9 - 0.03$	b_9
11	(.9,1.)	$b_9 - 0.02$	$b_9 - 0.02$	b_9^*

$b_7^* = b_7 + (1-b_7^2)/\{10(1 - ([10 \cdot b_7]/10)) + 1\}$.

$b_9^* = b_9 - (1 - b_9^2)/10$.

$b_7' = b_7 + (1 - b_7^2)/10$.

Table VIII. Estimator Performance Results for Cell 4.

Estimator	$E(\hat{\beta})$	$Var(\hat{\beta})$	Bias	Mean Abs. Error	MSE	MSPE
1	.25119	.00106	-.05846	.15335	.03767	1.06239
2	.27910	.00131	-.03055	.15061	.03533	1.05704
3	.26730	.00133	-.04235	.15065	.03568	1.05787
4	.27166	.00898	-.03799	.16570	.04348	1.06775
5	.23738	.00089	-.07227	.15609	.03967	1.06678
6	.27988	.00678	-.02977	.15942	.04008	1.06005
7	.25055	.00083	-.05910	.15291	.03749	1.06240
8	.24399	.00087	-.06565	.15449	.03857	1.06448
9	.28994	.00203	-.01971	.15006	.03465	1.05433
10	.23734	.00090	-.07230	.15613	.03969	1.06681
b_7^*	.35460	.00074	+.04495	.15712	.03599	1.05083
b_7^* -.04495	.30965	.00074	0.0	.14997	.03397	1.05265
b_9 + 0.05	.33994	.00203	+.03029	.15393	.03518	1.05058

absolute error value of 0.14997 and a MSE value of 0.03397, both slight improvements over estimator 9. A constant correction for the negative bias (-0.01971) of b_9 reduced the MSPE of b_9. However, over-corrections resulting in positive bias continued to reduce MSPE even more. The MSPE appeared to be near-minimum with a value of 1.05058 for the estimator $b_9 + 0.05$. Hence, 0.05 was used as the constant correction factor.

CHAPTER VI

VALIDATION OF ADAPTIVE ESTIMATORS

In order to assess the improvement in performance offered by the two types of adaptive estimators, additional Monte Carlo validation runs were made. Three different methods of data generation were used in order to afford different types of validation comparisons. For each of these types of comparison, the performance of the two types of adaptive estimators was compared with the performance of some or all of the standard estimators.

For the first method of data generation, β was drawn randomly from the interval (0,1). Using this β as the true parameter value, a series of length 40 was generated, parameter estimates were computed based on the first 20 terms of the series, 20 one-step predictions of the last 20 terms of the series were made, and statistics were accumulated. Again, this process was replicated 10000 times. For the second method of data generation, performance validation runs were made for 0.1 length parameter subintervals of the interval (0,1). For each of these runs, β was drawn randomly from a subinterval, a replication performed as described above, and the process repeated so as to acquire 10000 replications in that subinterval. The third method of data generation was analagous to that of the simulation runs discussed in Chapter IV. For these runs, 10000 replications were performed using each of several fixed β values.

Throughout the remainder of this chapter, we will discuss the performance comparison results of these validation runs for each of the three methods of series generation described above.

Interval (0,1) Comparisons

For the first method of data generation, where β was randomly drawn from the interval (0,1), the two types of adaptive estimators were compared with all eleven of the standard estimators considered in Chapter IV. A set of statistical multiple comparisons were computed for each of the three criteria of mean absolute error, MSE, and MSPE. Here each of these sets involved 78 simultaneous pairwise comparisons. Again, Bonferroni's method was used to find the critical value for testing at the $\alpha = 0.01$ significance level. From these tables of multiple comparison values (of the form of Table III, Chapter IV), Table IX was constructed. Table IX shows performance ranking of each estimator, along with statistical class groupings of equivalent performance, for each criterion. In this table, any two estimator numbers not underscored by the same line have significantly different means for the given criterion. Any two estimator numbers underscored by the same line are not significantly different. The estimator numbers in each case are arranged from left to right in ascending order of the criterion averages.

The results displayed in Table IX indicate good performance characteristics for the adaptive estimators. The

Table IX. Estimator Performance Groupings for First Type of Validation Run (β Drawn from the Interval (0,1)).

Mean Abs. Error	A2	A1	9	11	7	8	1	5	10	3	2	4	6
MSE	A2	A1	9	11	7	8	5	10	1	3	2	4	6
MSPE	A2	9	A1	11	7	3	1	8	2	6	4	5	10

A1 represents the first type of adaptive estimators based on the use of standard second-step estimators.

A2 represents the second type of adaptive estimators based on the use of ad hoc second-step estimators.

second type of adaptive estimator, A2, based on ad hoc second-step estimators, is statistically better than any of the other estimators for mean absolute error and MSE. For these two criteria, the first type of adaptive estimator, A1, based on standard second-step estimators is second best, and along with estimator 9, forms a class which is statistically better than all other standard estimators. The results for MSPE are not as pleasing. Even though the type-two adaptive estimator does result in the smallest MSPE, this value is not significantly smaller than that of several other estimators.

Some of the numerical results from this first type of validation run are given in Table A-V. This table shows each criterion mean and standard error for each estimator. These numbers formed the basis of the multiple comparisons summarized in Table IX. A look at some of the means given in Table A-V illustrates the order of magnitude of difference between estimator performance. For instance, consider the criterion of MSE. The difference in MSE performance of the two traditional estimators OLS and MLE was only 0.0011. The largest improvement over OLS offered by any standard estimator was 0.0055 by estimator 9, which is roughly the same as the improvement over OLS offered by estimator A1. However, the MSE improvement over OLS offered by estimator A2 was 0.0128, which is twice as large as the differences between OLS and A1 or 9, and over ten times as large the difference between OLS and MLE performance.

Subinterval Comparisons

For the second method of data generation, comparisons were made in each 0.1 length subinterval of (0,1). These comparisons involved criterion means and standard errors for each estimator in each subinterval. These means and standard errors for the criterion of MSE are shown in Table A-VI. The performance of the two types of adaptive estimators was only compared with a selected few of the best other estimators in each subinterval. Based on the fixed-parameter results of Chapter IV, only those estimators which appeared in the "best" group for some criterion at the parameter values which constitute the endpoints of a subinterval were selected for comparison in that subinterval. This resulted in varying numbers of pairwise comparisons for the different subintervals. Again, simultaneous multiple comparisons were made at the $\alpha = 0.01$ level of significance using Bonferroni's method. The results of these comparisons were used in the construction of Table X. This table shows the three best performing estimators for each criterion in each subinterval. The two types of adaptive estimators again perform well overall, but do not outperform all of the standard estimators in the subintervals close to one.

Comparisons at Fixed β Values

The third method of data generation required one simulation run analagous to that using fixed β values discussed

Table X. Best Performing Estimators in Subintervals of (0,1) - Second Type of Validation Run.

	Subinterval									
	(0,.1)	(.1,.2)	(.2,.3)	(.3,.4)	(.4,.5)	(.5,.6)	(.6,.7)	(.7,.8)	(.8,.9)	(.9,1)
Mean Abs. Error	10	A2*	A2*	A2*	A2*	A2*	A2*	9	9*	6
	5	A1	A1+	A1+	A1+	A1	9+	A2	11	3
	A1	10	10	9	9	9	A1	A1	A1	2
MSE	10	A2*	A2*	A2*	A2*	A2*	A2*	9	9	6
	5	A1	A1+	A1+	A1	A1	9	A2	3	2
	A1	10	9	9	9	9	A1	A1	A2	3
MSPE	10	A2	A2	A2	A2	A2	A2	A2	A2	6
	5	10	9	9	9	9	9	A1	A1	2
	8	5	10	A1	A1	A1	A1	9	9	3

A1 represents the first type of adaptive estimators.

A2 represents the second type of adaptive estimators.

* indicates estimator is significantly better than all others.

\+ indicates estimator is significantly better than all others, excluding the best one.

in Chapter IV. On each replication of this run, series were generated using each of the twelve parameter values used in the standard estimator comparisons of Chapter IV. The results of the average performance over 10000 replications were used for the comparisons. As an example of these performance results, means and standard errors for each estimator at fixed parameter values are given in Table A-VII for the criterion of MSE. For each parameter value, the performance of the two types of adaptive estimators was compared with the performance of the estimators in the "best" group for some criterion at that parameter value. As before, simultaneous multiple comparisons were made at the $\alpha = 0.01$ level of significance. These comparison results were summarized by listing the three best-performing estimators for each criterion at each parameter value, as shown in Table XI. As would be expected, performance results at parameter values as illustrated in Table XI are very similar to performance results for subintervals associated with those parameter values shown in Table X.

Table XI. Best Performing Estimators at Fixed β Values from Third Type of Validation Run.

	β Value											
	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95	.99	.999
Mean Abs. Error	5	A2*	A2*	A2*	A2*	A2*	A2	9*	9	6	6	6
	10	A1	A1+	A1+	A1	9	9	A2	11	3	2	2
	A2	5	9	9	9	A1	A1	A1	3	2	3	3
MSE	A2*	A2*	A2*	A2*	A1	A2*	A2	9	6	6	6	6
	10	A1	A1+	A1+	9	9	9	A2	9	2	2	3
	5	5	9	9	A2	A1	A1	A1	3	3	3	2
MSPE	10	A2	A2	A2	A2	A2	A2	A2	A2	6	6	6
	5	10	9	9	9	9	9	A1	A1	2	2	2
	8	5	5	A1	A1	A1	A1	9	9	3	3	3

A1 represents the first type of adaptive estimators.

A2 represents the second type of adaptive estimators.

* indicates estimator is significantly better than all others.

\+ indicates estimator is significantly better than all others, excluding the best one.

CHAPTER VII

SAMPLE SIZE SENSITIVITY AND LIMITATIONS OF THIS RESEARCH

Sample Size Sensitivity

Since in the development of the adaptive estimators a sample size of 20 was used throughout, some additional analysis was done to check the sensitivity of adaptive estimator performance to a change in sample size. As the sample size was increased with a resulting reduction in estimator bias, it was felt that the ad hoc correction factors used in construction of the second type of adaptive estimators might become ineffective. Hence, some additional Monte Carlo runs were made using sample sizes of 50 and 100 to investigate this sample size sensitivity.

Each of the runs for sample sizes 50 and 100 were validation runs of the same type as the first validation run discussed in Chapter VI. That is, for each replication, β was drawn randomly from the interval (0,1) and, using this β, a series of the specified length was generated. Since the sampling variances of the quantities compared in these runs were much smaller for the larger sample sizes, only 1000 replications were used on each run.

A few results from these validation runs for sample sizes 50 and 100 are shown in Table XII. In column one, empirical mean absolute error, MSE, and MSPE are shown for

Table XII. Some Estimator Performance Results Showing Sample Size Sensitivity of Adaptive Estimators.

	Best Standard Estimator (b_9)	1st Type of Adaptive Estimators (A1)	2nd Type of Adaptive Estimators (A2)	
n = 50			1.*	2.+
Mean Abs. Error	0.09089	0.09084	0.08912	0.08343
MSE	0.01435	0.01442	0.01356	0.01211
MSPE	1.01441	1.01445	1.01253	1.01129
n = 100				
Mean Abs. Error	0.06050	0.06055	0.07329	0.05785
MSE	0.00633	0.00634	0.00911	0.00568
MSPE	1.02064	1.02068	1.02433	1.02016

* using constant modifications.

\+ using modifications expressed as functions of the sample size.

estimator 9, which was the best performing standard estimator for each of the three criteria at each sample size. We can see that the performance results of the first type of adaptive estimators, given in column two, are very nearly the same as those of estimator 9 for each criterion at each sample size. This is similar to the validation performance results using sample sizes of 20, in that the performance of estimator 9 and that of the first type of adaptive estimators were not statistically different for either criterion. However, the performance of the first type of adaptive estimators was usually slightly worse than that of estimator 9 in these runs, where for a sample size of 20 it was usually slightly better. This fact seems to indicate a slight change in the relative performance of some of the estimators which make up the first type of adaptive estimators, or possibly just a difference in the results of using only 1000 replications.

In order to investigate sample size sensitivity of the second type of adaptive estimators, validation runs were made with sample sizes of 50 and 100 using the same second step estimators developed for sample size 20, as shown in Table VII. Of course, these ad hoc second-step estimators included the same constant modifications which were developed for sample size 20. The results of these runs, given in column 3 of Table XII, show that the ad hoc adaptive estimators still performed better by all three criteria than estimator 9 or the first type of adaptive estimators for

sample size 50. However, for sample size 100, the ad hoc corrections seemed to become inappropriate and we see that the performance of the ad hoc adaptive estimators became worse than that of estimator 9 and the first type of adaptive estimators for each of the three criteria. These results indicated the need to express the ad hoc corrections as functions of the sample size in order to have appropriate correction terms.

To determine the nature of appropriate correction terms, several exploratory simulation runs were made where, for sample size n, ad hoc corrections were expressed in terms of $\sqrt{n}$, n, and n^2. For instance, a constant c_1 was replaced by the term $c_2/\sqrt{n}$, where $c_2 = \sqrt{20}\ c_1$, and other constants similarly expressed in terms of $\sqrt{n}$ for one run. Analagously, c_1 was replaced by c_2/n, where $c_2 = 20\ c_1$ for some other runs, and similarly for n^2. In addition, the three ad hoc estimators b_7', b_7^*, and b_9^* described in Chapter V were expressed as functions of n. For $b_7' = b_7 + (1-b_7^2)/10$, the constant 10 was replaced by n/2, which gave a smaller adjustment to b_7 for larger sample sizes. Similarly, for $b_9^* = b_9 - (1-b_9^2)/10$, the 10 was replaced by n/2. For $b_7^* = b_7 + (1-b_7^2)/c^*$, the c^* term in the denominator was multiplied by n/20, which again accomplished reduction in the size of the adjustment term for larger sample sizes.

The best ad hoc adaptive estimator performance observed in the various runs resulted from the use of b_7', b_7^*, and b_9^* as discussed above, and the use of constant adjustments

expressed as functions of $\sqrt{n}$. The results of these runs are shown in column four of Table XII. We can see for n = 50 that the improvement in performance of the sample-size-dependent ad hoc estimators over estimator 9 was much larger for all three criteria than was the improvement offered by the adaptive estimators which rely on constant modifications. Also, for n = 100, the sample-size-dependent ad hoc estimators performed well. They resulted in lower values for all three criteria than either estimator 9 or the first type of adaptive estimators.

Thus, in conclusion, we can see that the second type of adaptive estimators, as developed, were sensitive to changes in sample size. However, the expression of the ad hoc modifications used in these estimators as appropriate functions of the sample size did result in adaptive estimators which performed well throughout a range of larger sample sizes.

Limitations of This Research

As is usually true of any study of narrow scope, this study has several limitations. First, it is limited in the sense that the AR(1) model is a simple subclass of ARIMA models. The nature of the investigations do not allow them to be generalized for parameter estimation for other types of ARIMA models, per se. In addition, aspects of modeling such as model identification and diagnostic checking for model improvement are not considered here.

It is assumed that the results of this study will be applied in parameter estimation of a series which is stationary and for which the AR(1) model has been identified, or for which information about the generating process suggests fitting of an AR(1) model. In practice this might require taking first or second differences of the raw data to achieve stationarity. In fact, in model identification, a series is differenced if preliminary parameter estimates are less than but not significantly different from one, as well as when they are greater than one. In the Monte Carlo simulations, series were generated by a process which was known to be stationary AR(1). However, no model identification and no differencing were done. So for these short series, estimates might have been greater than one and hence constrained to one by the truncation rule, or less than one but not significantly different from one, and still the fitting of an AR(1) model was "forced" on the data, and these AR(1) parameter estimates included in the accumulation of run statistics. This involved some model misspecification which may have, in some manner, biased the results of the simulations. In particular, results for the larger parameter values might be different from results which would have been acquired by considering only those parameter estimates of series for which sample statistics clearly indicated stationary AR(1) series.

Another possible limitation in the appropriateness of application of the Monte Carlo simulation results comes from

the choice of fitting the one-parameter AR(1) model

$$x_t = \beta x_{t-1} + \varepsilon_t \tag{7.1}$$

rather than the two-parameter AR(1) model

$$x_t = \beta_0 + \beta_1 x_{t-1} + \varepsilon_t ,$$

which reparameterized in terms of the process mean μ is given as

$$(x_t - \mu) = \beta(x_{t-1} - \mu) + \varepsilon_t . \tag{7.2}$$

Since in the simulations the generating process used $\mu = 0$, it seemed appropriate to fit model (7.1). However, for β values close to one, and short series, the sample mean may be considerably different from zero. Gonedes and Roberts [16] found that the fitting of model (7.1) to short series generated by equation (7.1) using OLS estimation, resulted in smaller bias, MSE, and MSPE (of 20 one-step predictions) in estimating β than did the fitting of model (7.2) to the same data, where μ was also estimated from the sample data. The question then arises as to how this reduction in bias, MSE, and MSPE of not estimating the process mean influenced the relative performance results of the estimators of β considered in this dissertation, and hence, the conclusions that are drawn. In this respect, it is unclear in application of the adaptive estimators in the estimation of a given series what effect the estimation of the mean of the series might have on the performance of the adaptive estimators in estimating β for the mean-adjusted series.

One other possible limitation of this study is the choice of Burg's estimator, b_7, as the first-step estimator

in developing the adaptive estimators. As was discussed in Chapter V, the good overall performance of estimator 9 in the fixed parameter studies of Chapter IV would suggest it as a good first-step estimator. However, since estimator 9 was used as a possible second-step estimator in each cell, it is not clear whether the use of this estimator as the first-step estimator would enhance the overall performance of the two-step adaptive estimation procedures.

CHAPTER VIII

APPLICABILITY OF THIS RESEARCH AND CONCLUDING REMARKS

In this concluding chapter, three aspects of the applicability of this research will be discussed. First, based on the findings of this study, a procedure for selecting an appropriate estimator for use in an applied estimation will be given. Next, several examples from the literature pertaining to parameter estimation and forecasting of AR(1) series will be discussed in an effort to illustrate potential areas of applicability of the adaptive estimators developed in this dissertation. Also, the results of the application of the adaptive estimators in the actual estimation and forecasting of a consumer price index series will be shown. Of course the small average improvement in absolute bias, MSE, and MSPE performance offered by the adaptive estimators certainly does not guarantee that improved performance can be observed for any one given series. Nevertheless, this example will serve to demonstrate the application of these estimators.

Recommendations for Applied Estimation

Based on the results of the validation runs discussed in Chapter VI, certain recommendations for applied estimation can be given. If one is faced with parameter estimation of a series for which an AR(1) model has been

specified, but for which little or no information as to the true β value is available (more specifically β is assumed uniformly distributed in (0,1) in the Bayesian sense), one needs to first refer to Table IX. For a given criterion, the selection of an estimator will depend on the performance ranking shown in Table IX and the estimator means and standard errors shown in Table A-V. The selection of an estimator rests on one's subjective evaluation of the tradeoff between performance gains and computational difficulty of the various estimators.

For example, suppose one wishes to estimate β and the criterion of interest is MSE. A look at Table IX would seem to indicate the use of estimator A2, or possibly of estimators A1 or 9. The middle column of Table A-V shows the MSE values for A2, A1, and 9 to be 0.0263, 0.0333, and 0.0336 respectively, while the MSE values for OLS and MLE estimators are 0.0391 and 0.0380 respectively. The 0.0070 difference in performance of A2 over A1, when standard errors are of the order 0.0005, together with the fact that the difference between MLE and OLS is of the magnitude 0.0009, would suggest the use of A2. However, if estimator A2 was thought to be computationally difficult, then either estimator 9 or A1 could be used. Superficially it might seem that A1 is a much more cumbersome or computationally difficult estimator to use in practice as compared to any simple standard estimator, such as estimator 9 or estimator 1 (OLS). However, it should be emphasized, by noting Table VI, which defines estimator A1, that the computation of estimator A1 only

requires the computation of estimators b_3, b_7, and b_9, each of which requires virtually the same quantities for its computation and thus could easily be computed each time the use of estimator A1 is considered. This allows the practical application of estimator A1 to be a one-step procedure computationally, and requires only a few additional arithmetic operations compared to the computation of the OLS estimator. On the other hand, if the criterion of interest had been MSPE, the information in Table IX and Table A-V suggests that one might be indifferent among the choice of the estimators A2, 9, A1, 11, 7, 3, and 1.

Suppose however, that in a practical estimation situation one has some fairly strong a priori information that the true β value lies in some subinterval of the interval (0,1). In this case, recommendations for estimator choice rely on the information given in Table X and Table A-VI. For example, suppose one feels that the true β is in the interval (.2, .6), and is interested in good MSE performance. Then Table X suggests the use of A2, A1, or 9. Examination of Table A-VI shows estimator A2 to have smaller MSE than A1 by roughly 0.01 to 0.02 (where standard errors are in the range 0.0003 to 0.0005) throughout the interval (.2, .6). This represents a MSE reduction of 30% to 40% for A2 over A1, which would strongly suggest the use of A2. A1 offers an improvement over estimator 9 of 0.002 to 0.003 for most of the range (.2, .6), and might reasonably be used in this case if A2 were not used.

The fixed parameter performance results summarized in Table XI and the MSE means and standard errors given in Table A-VII are of no practical value in the selection of an appropriate estimator for use in an application. They are included in this dissertation to facilitate comparison to some previous studies which reported their performance results only in terms of fixed β values.

Review of Examples

Many articles found in the economic and finance literature discuss studies which include some type of ARIMA modeling. In a number of these studies AR(1) models were used in describing many different types of series. In most of these studies the validity of the conclusions that were drawn depend on the accuracy of parameter estimation and/or prediction. Often these studies involve short series, and quite frequently OLS estimation was used. In many of these cases, potential for improved performance could be offered by the use of one of the adaptive estimators.

Several articles pertain to the modeling of stock prices and commodity futures prices [5,8,22,35]. These articles mainly address the question of whether these series are random walks, or large parameter AR(1). Here parameter estimation accuracy (mean absolute error and MSE) is of crucial importance. Many other articles pertain to the study of economic series such as annual or quarterly GNP, annual velocity of money, and short term interest rates [13,17,28,29].

For example, Nelson illustrates the fitting of several of these economic series in his study of the Federal Reserve Board-MIT-Penn econometric model of the U.S. Economy [28,29]. In this analysis he models 14 endogenous variables of the FRB-MIT-Penn model with ARIMA models. Of these 14, real GNP, GNP deflator-price index, and consumer goods price index are modeled as AR(1) in their first differences. He concludes that composite forecasts based on a combination of the individual ARIMA forecasts and the econometric model forecasts rely significantly on the ARIMA forecasts for 10 of the 14 variables. This suggests that the ARIMA predictions do embody information available in the history of individual series which is not utilized by the FRB-MIT-Penn model. In this respect, good AR(1) series prediction can be important as an alternative to certain findings of other econometric models or at least serve as a benchmark for their evaluation.

The accounting literature has numerous articles [1,2,11,14,18,32,37] discussing the time series properties and modeling of earnings, earnings-per-share, and other income numbers. Of particular interest to us because of the fitting of numerous AR(1) models (often based on short series) are the articles [1], [11], and [37].

Watts and Leftwich [37] investigated whether the use of Box-Jenkins techniques on annual earnings (available for common) resulted in models with better predictive ability than random walk. They investigated 32 firms in three

industries (railroads, petroleum, and metals). They identified AR(1) models for all but one of the ten railroads, five of the eleven petroleum firms, and two of the eleven metal industries, based on 38 years of data. In order to investigate sample size influence they also used periods of 50, 55, and 60 years for each firm which had at least 60 years of data. They concluded that approximately half of the processes they modeled were significantly different from random walk. However, the one-step-ahead predictive ability of these fitted models seemed in most cases to be no better than random walk or random walk with trend. Also, the large number of model specification changes with changing sample sizes seemed to imply structural change and/or model misspecification problems.

Albrecht, Lookabill, and McKeown [1] investigated both nondeflated (earnings available to common stockholders) and deflated (earnings available to common stockholders/stockholders equity of previous period) earnings for 49 firms in three industries (foods, chemicals, steel) based on a 25 year estimation period. They fit Box-Jenkins models, and compared to random walk and random walk with drift. For the nondeflated data, they fit several AR(1) models for steel firms, concluding the steel industry tended to be autoregressive. However, the chemical industry tended to exhibit random walk behavior and the food industry results were mixed. Deflated earnings in all three industries were suggestive of random walk models. In all cases the

predictive ability of the Box-Jenkins models appeared to be no better than that of the best of the two random walk models.

An interesting study by Dopuch and Watts [11] dealt exclusively with 11 steel firms. They attempted to evaluate the significance of an accounting change (from straight-line to accelerated depreciation) based on the evaluation of the time series characteristics of net income. Here small sample estimation was crucial because the authors only had approximately 30 years of data before the change and 11+ years after the change for most firms. In fitting Box-Jenkins models to before-change series, they found that 5 of the 11 firms could be modeled as AR(1). They then fit the same respective model to each after-change series and estimated the parameters. They found that the accounting switch had a significant effect on the income process for 8 of the 11 firms (3 of the 5 AR(1) parameters).

Thus in each of the above studies dealing with earnings series, we see the reliance of the conclusions that are drawn on accurate parameter estimation and forecasting. In most of these studies, the use of the adaptive estimators at some points might offer a potential improvement in estimation and prediction accuracy and hence in the validity of the conclusions drawn.

An Example: Forecasting a Consumer Price Index Series

As an illustration of the application of the adaptive estimators, some one-step forecasts were made using the

consumer price index series. The data used was acquired from the National Bureau of Economic Research data tapes. The particular series used consisted of monthly observations of seasonally adjusted values of the consumer price index for all items with 1967 as the base year.

A plot of the first 312 observations of this series, form January 1947 through December 1972, showed the series to be nonstationary. After first differences were taken, preliminary model identification and diagnostic checking indicated the differenced series to be adequately modeled as an AR(1). Subsamples of this series of length 40 were chosen for which sample fingerprints still indicated the adequacy of the AR(1) model in describing the data. One such series of length 40 was the period from September 1948 through December 1951.

To illustrate the application and performance of the standard estimators as well as the two types of adaptive estimators, initial parameter estimates were computed using the first 20 terms of this series. Using these estimates, one-step-ahead predictions were made and prediction errors computed for each estimator. Then, the second through twenty-first terms were used to compute parameter estimates, and one-step forecasts made for the twenty-second term. In this manner, terms 21 through 40 in the first-differenced series were forecast. In computing the results that are shown in this section, it should be noted that the sample mean of each sample of 20 terms was subtracted from each

term before parameter estimation and forecasting, so that in effect, the two-parameter model was fit. However, the same run made without adjusting for the mean, and hence, fitting the one-parameter model resulted in larger parameter estimates for all estimators on most of the 20-term samples, but very nearly the same results for MSPE.

The results of the MSPE performance for the standard estimators, as well as the two types of adaptive estimators, are given in Table XIII. In this table we can see that the ad hoc adaptive estimator A2 has the smallest MSPE, while the first type of adaptive estimator A1 has the second smallest MSPE. Predicting terms 21 through 40 of the differenced series allows for the forecasting of terms 22 through 41 of the original series. These twenty one-step forecasts using the ad hoc adaptive estimator A2 are plotted along with the actual values of the series in Figure 1. The good forecasting performance in this example in terms of the criterion of MSPE illustrates the potential applicability of this estimator for this criterion.

Concluding Remarks

Through the use of Monte Carlo comparison studies, we have given a more complete catagorization of the small sample performance of several AR(1) estimators for the criteria of bias, mean absolute error, MSE, and MSPE than has been given before. In addition, we have shown the development of two types of adaptive estimators and validated the efficacy of their performance. The approach used in the development

Table XIII. Results from Estimation and Prediction of Consumer Price Index Series Showing MSPE for Twenty One-Step Predictions.

Estimator	MSPE
1	0.18788
2	0.18857
3	0.21579
4	0.20733
5	0.19230
6	0.19037
7	0.19640
8	0.18918
9	0.18838
10	0.19250
A1	0.18536
A2	0.18238

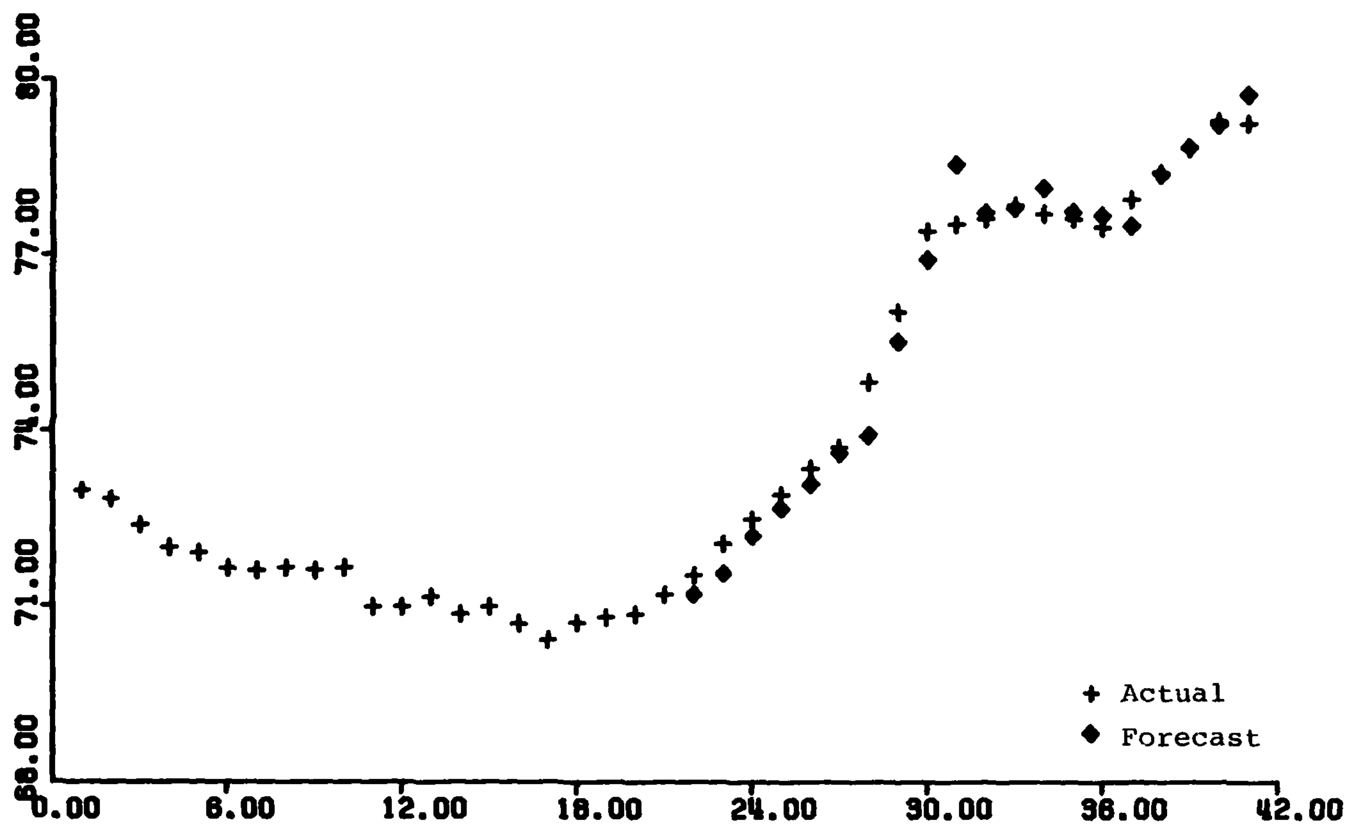

Figure 1. Consumer Price Index Series and Forecasts

of these estimators shows promise for use in additional investigations. Possibilities for investigation include using a different first-step estimator, studying performance based on the fitting of the two-parameter model, including model identification in the study of generated series, and incorporating a more thorough study of sample size dependence. Additional areas for future research include better characterization of the sampling distributions of the estimators compared in this study, and the development of a procedure for obtaining confidence intervals for parameter estimates.

APPENDIX

TABLES OF ESTIMATOR COMPARISON RESULTS

Table A-I. Estimator Performance Groupings at Fixed β Values for the Criterion of Bias.

β = .1	4	3	2	6	11	1	7	8	10	5	9
β = .2	6	2	4	3	1	11	7	8	9	10	5
β = .3	2	6	4	9	3	11	1	7	8	10	5
β = .4	9	2	6	4	3	11	1	7	8	5	10
β = .5	6	2	4	9	3	11	1	7	8	10	5
β = .6	6	2	4	3	9	11	1	7	8	10	5
β = .7	6	2	4	3	9	11	1	7	8	5	10
β = .8	6	2	3	4	9	11	1	7	8	5	10
β = .9	6	2	3	4	9	11	1	7	8	5	10
β = .95	6	2	3	4	9	11	7	1	8	5	10
β = .99	6	2	3	4	9	11	7	1	8	5	10
β = .999	6	2	3	9	7	11	4	1	8	5	10

Table A-II. Estimator Performance Groupings at Fixed β Values for the Criterion of Mean Absolute Error.

β = .1	5	10	8	9	1	7	11	3	2	4	6
β = .2	10	5	9	8	7	11	1	3	2	4	6
β = .3	9	5	10	8	7	11	1	3	2	4	6
β = .4	9	10	5	8	7	11	1	3	2	4	6
β = .5	9	10	5	8	11	7	1	3	2	4	6
β = .6	9	11	7	8	1	10	5	3	2	4	6
β = .7	9	11	7	8	1	3	5	10	2	4	6
β = .8	9	11	7	3	1	8	4	10	5	2	6
β = .9	9	11	3	7	6	2	4	1	8	5	10
β = .95	6	3	2	9	11	7	4	1	8	5	10
β = .99	6	2	3	11	9	4	7	1	8	5	10
β = .999	6	2	3	9	11	7	4	1	8	5	10

Table A-III. Estimator Performance Groupings at Fixed β Values for the Criterion of MSE

β = .1	10	5	8	9	7	1	11	3	2	4	6
β = .2	5	10	9	8	7	11	1	3	2	4	6
β = .3	9	5	10	8	7	1	11	3	2	4	6
β = .4	9	10	5	8	7	11	1	3	2	4	6
β = .5	9	10	5	8	7	1	11	3	2	4	6
β = .6	9	11	7	8	1	10	5	3	2	4	6
β = .7	9	11	7	3	1	8	5	10	2	6	4
β = .8	9	3	11	7	2	1	6	8	4	10	5
β = .9	6	9	3	2	11	7	4	1	8	5	10
β = .95	6	2	3	9	11	4	7	1	8	5	10
β = .99	6	2	3	9	11	4	7	1	8	5	10
β = .999	6	2	3	9	11	7	4	1	8	5	10

Table A-IV. Estimator Performance Groupings at Fixed β Values for the Criterion of MSPE.

β = .1	10	5	8	9	7	11	1	3	2	4	6
β = .2	10	5	9	7	8	1	11	3	2	4	6
β = .3	9	5	10	8	7	1	11	3	2	4	6
β = .4	9	5	10	8	7	11	1	3	2	4	6
β = .5	9	10	5	8	7	11	1	3	2	4	6
β = .6	9	7	11	8	1	5	10	3	2	4	6
β = .7	9	11	7	3	1	8	5	10	2	4	6
β = .8	9	11	3	7	1	2	8	6	4	10	5
β = .9	9	3	6	2	11	7	4	1	8	5	10
β = .95	6	2	3	9	11	4	7	1	8	5	10
β = .99	6	2	3	9	11	4	7	1	8	5	10
β = .999	6	2	3	9	11	7	1	4	8	5	10

Table A-V. Empirical Means (Standard Errors) from First Type of Validation Run Where β was Drawn from (0,1).

Estimator	Mean Abs. Error	MSE	MSPE
1	0.1498(.0013)	0.0391(.0007)	1.0667(.0039)
2	0.1637(.0013)	0.0446(.0007)	1.0706(.0038)
3	0.1532(.0013)	0.0406(.0007)	1.0649(.0038)
4	0.1676(.0014)	0.0484(.0008)	1.0783(.0040)
5	0.1513(.0013)	0.0391(.0006)	1.0830(.0044)
6	0.1686(.0014)	0.0486(.0008)	1.0766(.0039)
7	0.1468(.0013)	0.0381(.0006)	1.0647(.0039)
8	0.1484(.0013)	0.0384(.0006)	1.0696(.0040)
9	0.1389(.0012)	0.0336(.0006)	1.0561(.0037)
10	0.1513(.0013)	0.0391(.0006)	1.0837(.0044)
11	0.1461(.0013)	0.0380(.0007)	1.0628(.0039)
A1	0.1384(.0012)	0.0333(.0006)	1.0563(.0037)
A2	0.1258(.0010)	0.0263(.0004)	1.0477(.0036)

Table A-VI. Empirical Mean Squared Errors (Standard Errors) from Second Type of Validation Runs where β was Drawn from Subintervals of (0.1).

Estimator	Subinterval									
	(0,.1)	(.1,.2)	(.2,.3)	(.3,.4)	(.4,.5)	(.5,.6)	(.6,.7)	(.7,.8)	(.8,.9)	(.9,1)
1	481(6)[a]	475(6)	465(6)	449(6)	428(7)	402(7)	369(7)	329(7)	275(6)	185(5)
2	593(8)	584(8)	568(8)	542(7)	509(7)	468(7)	416(7)	347(6)	249(5)	122(4)
3	542(7)	534(7)	517(7)	493(7)	561(7)	420(7)	372(7)	312(6)	233(6)	125(4)
4	636(9)	628(9)	611(8)	586(8)	552(8)	505(8)	445(7)	368(7)	275(6)	165(5)
5	429(6)	427(6)	424(6)	418(6)	410(6)	399(7)	386(7)	369(7)	342(7)	269(6)
6	650(9)	641(9)	624(9)	597(9)	560(8)	511(8)	445(7)	359(6)	247(6)	116(5)
7	476(6)	470(6)	459(6)	443(6)	421(7)	393(7)	359(7)	317(7)	262(6)	170(5)
8	453(6)	449(6)	441(6)	430(6)	415(6)	395(7)	371(6)	340(7)	297(6)	215(6)
9	465(6)	442(6)	418(6)	392(5)	365(5)	334(5)	301(6)	263(5)	216(5)	142(4)
10	429(6)	427(6)	423(6)	418(6)	410(6)	399(7)	386(7)	369(7)	343(7)	270(6)
11	484(7)	487(7)	466(7)	448(7)	424(7)	393(7)	355(7)	309(6)	248(6)	155(5)
A1	447(6)	416(5)	386(5)	361(5)	344(5)	331(5)	317(6)	292(6)	249(6)	165(5)
A2	473(5)	306(4)	220(4)	200(3)	219(3)	248(3)	266(4)	268(5)	247(5)	176(4)

[a] Means and standard errors given have been multiplied by 10^4.

Table A-VII. Empirical Mean Squared Errors (Standard Errors) from Third Type of Validation Run Using Fixed β Values.

	β Value									
Estimator	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95
1	488(7)[a]	478(7)	463(7)	446(7)	425(7)	401(7)	360(7)	318(7)	245(6)	189(5)
2	602(8)	586(8)	563(8)	536(8)	500(8)	458(7)	394(7)	316(6)	191(5)	118(4)
3	548(8)	531(7)	514(8)	485(7)	452(7)	412(7)	354(7)	291(7)	188(5)	123(4)
4	645(9)	627(9)	606(9)	576(9)	539(9)	489(8)	419(8)	337(7)	231(6)	168(5)
5	435(6)	430(6)	424(6)	419(7)	414(7)	408(7)	387(7)	372(7)	325(7)	280(6)
6	669(10)	646(9)	622(9)	594(9)	551(9)	497(8)	418(8)	323(7)	187(5)	113(4)
7	481(6)	470(7)	457(7)	439(7)	417(7)	391(7)	349(7)	306(7)	228(6)	174(5)
8	459(6)	451(6)	440(6)	429(7)	414(7)	398(7)	366(7)	335(7)	273(6)	221(5)
9	462(6)	433(6)	408(6)	382(6)	356(6)	327(6)	288(6)	252(6)	187(5)	145(4)
10	435(6)	430(6)	424(6)	419(7)	414(7)	408(7)	387(7)	372(7)	325(7)	280(6)
11	490(7)	477(7)	464(7)	443(7)	419(7)	389(7)	343(7)	295(7)	212(6)	158(5)
A1	440(6)	404(5)	375(5)	354(6)	343(6)	334(6)	311(6)	284(6)	219(5)	170(5)
A2	384(4)	253(4)	203(3)	207(3)	233(3)	265(4)	275(5)	271(5)	227(5)	183(4)

[a] Means and standard errors given have been multiplied by 10^4.

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The small sample performance of several AR (1) estimators is investigated through the use of Monte Carlo comparison studies. The performance of these estimators is compared with respect to the criteria of bias, mean squared error, mean absolute error, and mean squared prediction error. Statistical performance groupings at various fixed parameter values from (0,1) are determined based on pair-wise multiple comparisons of estimator performance results.

Two types of two-step adaptive estimators are developed. One type relies on the use of only standard estimators, while the other type includes ad hoc

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modifications to standard estimators. The efficacy of performance of these estimators is validated through the use of additional Monte Carlo runs based on three different conditions of parameter selection for data generation. The sensitivity of these estimators to their use with larger sample sizes is also investigated.

Based on the various simulation results, recommendations regarding estimator selection for use in applied estimation are given. The applicability of the adaptive estimators is discussed and an example illustrating their application in forecasting an economic series is given.